An Inversion Technique for obtaining Quasi-Parabolic layer parameters from VI Ionograms

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Abstract— **Vertical incidence ionograms are obtained from sweep HF pulsed vertical incidence radar and provide information on the state of the ionosphere directly above the transmitter/receiver location. This paper describes a vertical incidence inversion technique where the ionospheric electron density profile is determined from the recorded layer echo traces found on vertical incidence ionograms. The vertical incidence inversion method is user friendly, in that the required inputs from the ionogram are readily obtainable. The inversion method is robust, in that the constraints within the method are flexible enough to accommodate the required input data from the ionograms and is able to home-in to the best possible solution.**

Index Terms— **ionogram, ionosphere, inversion, quasi-parabolic.**

I. INTRODUCTION

VERTICAL Incidence (VI) ionograms are the experimental results obtained from a swept frequency HF radar, where the received signals are reflected from a region of the ionosphere vertically above the sounder location. The ionosphere is the medium through which both the transmitted and received signals traverse and a VI ionogram contains useful information regarding the state of the ionosphere. Inversion methods are used to determine the ionospheric electron density or plasma frequency profile from the VI ionograms. This new inversion method is a fast and accurate inversion technique.

A mathematical representation of the vertical distribution of electron density of the ionosphere using quasi-parabolic segment, QPS, was developed by [1]. The QPS model also provides explicit equations to determine ray path parameters that are important for Over-The-Horizon radar and Beyond-Line-of-Sight communications. Until now there has been a problem in accurately inverting the VI ionosonde data to the layer parameters used in the QPS model.

Reference [2] and [3] developed a profile inversion technique. Their method inverted the virtual height data on the VI ionogram to the vertical electron density profile, described by a sum of Shifted Chebyshev polynomials where the coefficients are optimized in a least squares sense to the measured virtual height data. The method produces a monotonic profile with a single polynomial. A parabolic description of the ionosphere based on [1] is then obtained by fitting parabolic layers to the shifted Chebyshev profile. Thus, for a typical daytime ionosphere consisting of an E layer, F1 and F2 layers, there exist 9 parabolic parameters, 3 describing each of the ionospheric layers. Its critical frequency, peak height of the layer and the semi-thickness of the layer represent each ionospheric layer. Thus, $foF2, h_mF2$ and y_mF2 represent the respective F2 layer parameters.

A new technique described here does away with the need to firstly invert the virtual height data to a sum of shifted Chebyshev polynomials. This new inversion technique uses the layer echo traces on the VI ionogram directly in the inversion process. This new method is fast, robust and produces results in the form of the quasi-parabolic segment, QPS, ionospheric parameters.

A new VI inversion method must satisfy a number of criteria; Firstly, the method needs to be robust, and the constraints within the method must be flexible enough so as to accommodate data from actual vertical incidence sounding echo traces. The inversion method must be able to home in to the best possible solution and must be user friendly, in that the required inputs from the VI ionograms are readily obtainable.

This new inversion technique derives quasi-parabolic ionospheric layer parameters from the echo traces on the VI ionograms. At least 2 data points are required from each of the layer echo traces. The peak frequency of the layer echo traces, which is equivalent to the peak electron density of the ionospheric layer, is also a required input.

II. QPS - FORMALISM

The QP layer defined by [1] and [4] is given by

$$N_e = \begin{cases} N_m\left[1-\left(\dfrac{r-r_m}{y_m}\right)^2\left(\dfrac{r_b}{r}\right)^2\right], & r_b < r < r_m\left\{\dfrac{r_b}{r_b-y_m}\right\}, \\ 0, & otherwise \end{cases}$$

where N_e is the electron density at a radial distance r from the Earth's center, N_m is the maximum electron density at the radial height r_m, r_b is the radial base height of the ionospheric layer and y_m is the layer semi-layer thickness.

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Reference [4] showed that the model ionosphere could be built up using the QP model to describe each of the ionospheric layers and QP segments to smoothly join the QP layers together. They called this model the QPS model and showed the series of equations describing the ionosphere may be written as

$$N_E = a_E - b_E\left(1 - \frac{r_E}{r}\right)^2 \qquad \text{E layer}$$

$$N_j = a_E - b_j\left(1 - \frac{r_E}{r}\right)^2 \qquad \text{Joining layer}$$

$$N_F = a_F - b_F\left(1 - \frac{r_F}{r}\right)^2 \qquad \text{F layer.}$$

Where $a = N_m$ and $b = N_m \left(r_b / y_m\right)^2$ and

$$b_j = -r_F b_F\left(1 - \frac{r_F}{r_c}\right) \Big/ r_E\left(1 - \frac{r_E}{r_c}\right)$$

where the joining point, r_c, joining the joining layer to the F layer may be written as

$$r_c = \frac{r_F b_F\left(\frac{r_F}{r_E} - 1\right)}{a_F - a_E + b_F\left(\frac{r_F}{r_E} - 1\right)}$$

If a F1 layer were present then another joining layer between the F1 and F2 QPS layers would be required.

Reference [4] derived exact expressions for calculating ray parameters such as the virtual height, h', for propagation in a spherically stratified ionosphere consisting of a single QP layer. Where the equation for calculating the virtual height at vertical incidence may be written as

$$h' = r_b - r_\circ + \int_{r_b}^{r_t} \frac{rdr}{r\sqrt{Ar^2 + Br + C}} \qquad (1)$$

where r_t is the radial height at which the ray is reflected and

$$r^2\mu^2 = Ar^2 + Br + C$$

where μ is the refractive index.

Reference [1] expanded on their method to produce explicit equations for the ray parameters in a spherically stratified ionosphere for their model, where if the ray propagates through n segments the virtual height at vertical incidence may be written as

$$h' = r_b - r_\circ + \sum_n \left(U_n - L_n\right)$$

where U_n and L_n represent the values of the integral in equation (1) at the upper and lower bounds.

In general the more data points input into the inversion technique the more realistic the ionospheric layer parameters.

The inversion method described here is similar in design to the backscatter ionogram inversion method described by [5] and [6].

This new method of inversion of the VI ionogram echo traces requires the determination of the three layer parameters from each of the quasi-parabolic layers, namely critical frequency f_c, base height of the ionosphere r_b, and height of maximum electron density r_m. The critical frequency of each ionospheric layer is determined directly from the VI ionograms as it is equivalent to the peak frequency of the corresponding layer echo trace. Thus the inversion technique is only required to solve for 2 parameters, namely r_b and r_m for each layer.

The inversion technique begins with inversion of the VI ionogram E layer echo trace. Once the parameters for the E layer have been calculated the inversion technique begins on the F1 layer echo trace using the E layer parameter results as well as the QPS equations for the joining layer, which smoothly joins the E layer and the F1 layer. The process is repeated for the F2 layer where the layer parameters already determined for the E and F1 layers as well as the equations for the joining layer that joins the F1 and F2 layers are required in the inversion process.

The inversion technique requires n (where n is at least 2) data points from each of the layer echo traces, where the n virtual heights are chosen $h'_1, h'_2 \ldots h'_n$ corresponding to the n frequencies f_1, $f_2 \ldots f_n$ respectively. The method then sets out to find a set of values for the layer parameters, which yield to within a specified accuracy, to the data points chosen. To accomplish this an initial guess of the ray parameters (r_b, r_m) is made. Since we are dealing with VI ionograms the elevation angles of the swept frequency signal, are assumed to be 90° to the ground.

Using the assumed layer parameters together with the analytic expressions for the virtual height for the quasi-parabolic layer the virtual height can be determined. Let the computed virtual height, which most likeably will differ from the real values, $h'_1, h'_2 \ldots h'_n$ be $h'_{c1}, h'_{c2} \ldots h'_{cn}$, respectively, and let the corresponding differences between the real and computed values be $\Delta h'_1, \Delta h'_2 \;\ldots \Delta h'_n$, respectively. Then, to a first approximation the amount Δr_b and Δr_m by which the assumed layer parameters should be incremented so that $\Delta h'_1, \Delta h'_2 \;\ldots\; \Delta h'_n$, are a minimum is given by

$$\Delta h'(f) = \sum_i \frac{\partial h'}{\partial x_i} \delta\, x_i$$

or in the form

$$\begin{bmatrix} \Delta h'_1 \\ \Delta h'_2 \\ \vdots \\ \Delta h'_n \end{bmatrix} = \begin{bmatrix} \left(\dfrac{\partial h'}{\partial r_b}\right)_{f_1} & \left(\dfrac{\partial h'}{\partial r_m}\right)_{f_1} \\ \left(\dfrac{\partial h'}{\partial r_b}\right)_{f_2} & \left(\dfrac{\partial h'}{\partial r_m}\right)_{f_2} \\ \vdots & \vdots \\ \left(\dfrac{\partial h'}{\partial r_b}\right)_{f_1} & \left(\dfrac{\partial h'}{\partial r_m}\right)_{f_n} \end{bmatrix}^{-1} \begin{bmatrix} \Delta r_b \\ \Delta r_m \end{bmatrix}$$

or as $P = AX$

The inversion of this is

$$X = A^{-1}P$$

and maybe written as

$$X = (A_T A)^{-1} A_T P$$

where A_T is the transpose matrix of A and the square matrix $A_T A$ is suitable for inversion.

The assumed layer parameter values are then incremented by Δr_b and Δr_m. The entire procedure begins again with the new assumed values until the differences in $\Delta h'_1, \Delta h'_2 \ldots \Delta h'_n$, converge to a small-specified minimum, thereby obtaining the final solution of the layer parameters.

Once the layer parameters have been calculated the next layer parameters are then solved for using the technique above and the layer parameters already determined. For example, let us assume, that the E layer parameters r_{bE}, r_E, foE have already been evaluated using the method shown above. The peak frequency of the F1 layer, $foF1$, may be determined directly from the VI ionogram. The unknowns to be solved for are r_{bF1} and r_{F1}. Two additional QP segments are now involved where one represents the joining layer, which smoothly joins the E and F1 layers and the QPS layer representing the F1 layer from the peak of the F1 layer down to the point where these to layers are smoothly attached. The joining layer is made up of the E and F1 layer parameters (refer to [4]). The ray enters 2 or 3 QP segments to get to the F1 layer, thus 3 QP segments are required to determine the F1 layer parameters using the inversion technique shown here. The equation for the virtual height in determining the F1 layer parameters contains the parts of the ray path, in free space, in the QPS representing the E layer, in the joining QPS layer and in the QPS representing the F1 layer. The equation for the virtual height for ray paths being reflected from the F1 layer may be written as

$$h' = r_b - r_\circ + U_E - L_E + U_j - L_j + U_{F1} - L_{F1}$$

Once the F1 layer parameters are known the F2 layer parameters maybe calculated. Hence, 5 QP segments are required to determine the F2 layer parameters.

III. INVERSION OF A SYNTHESIZED VI IONOGRAM

In order to test the accuracy and the robustness of this method, a synthesized profile with known QPS parameters was used. With the aid of analytic ray tracing programs, synthesized VI ionograms were determined and the inversion technique was then applied.

In this example the synthesized VI ionogram, in Figure 1, was chosen, having the layer parameters

$$foE = 3.2\,\text{MHz},\ y_m E = 14\ \text{km},\ r_E = 104 + r_0\ \text{km}$$
$$foF1 = 4.61\,\text{MHz},\ y_m F1 = 77\ \text{km},\ r_{F1} = 183 + r_0\ \text{km}$$
$$foF2 = 6.95\text{MHz},\ y_m F2 = 130\ \text{km},\ r_{F2} = 283 + r_0\ \text{km}$$

where r_0 = 6370 km (the radius of the Earth).

In practice data points would be chosen from each of the layer echo traces in Figure 1. Where at least 2 points, from each of the layer echo traces, are required. The peak frequency from each of the layers is also a required input.

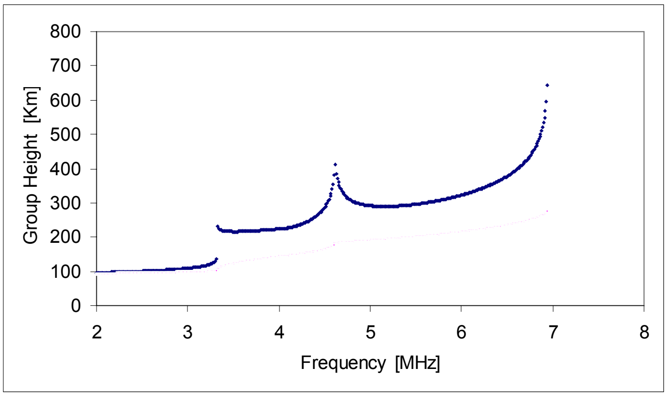


Fig. 1. Synthesized VI ionogram, representing a typical daytime ionosphere.

In this case since we are testing the accuracy of the technique and thus greater accuracy in the input data is required and Table 1 shows the input data points chosen from each of the layer echo traces. Only 2 data points were chosen from each layer echo trace. In general the more data points the higher the accuracy.

TABLE I
INPUT DATA POINTS

Data point E layer	Frequency [MHz]	Group Height, h' [km]	$\Delta h'$ (20 iter.) [km]
1	2.1	96.59285	2.6×10^{-5}
2	3.0	108.8607	3.3×10^{-5}
Data point F1 layer			
1	4.1	226.7465	3.2×10^{-5}
2	4.5	286.4045	3.7×10^{-6}
Data point F2 layer			
1	5.0	291.6644	2.6×10^{-5}
2	6.0	322.6815	5.4×10^{-5}

The 4th column in Table I show the difference between the computed and inputted group height data after 20 Iterations of the inversion program. The difference in group height is <<1meter.

Due to the conditions on the QPS model an iterative approach on the initial guess is normally required. In our program an iterative procedure is performed so that no guessing of the initial layer parameters is required. The inversion program uses an iterative approach to choose a first guess to satisfy conditions and to add robustness to the technique.

The computed QPS layer parameters were determined using the inversion technique and found to be equal to the true layer parameters. Fig. 2 shows the QPS ionospheric profile having the layer parameters above.

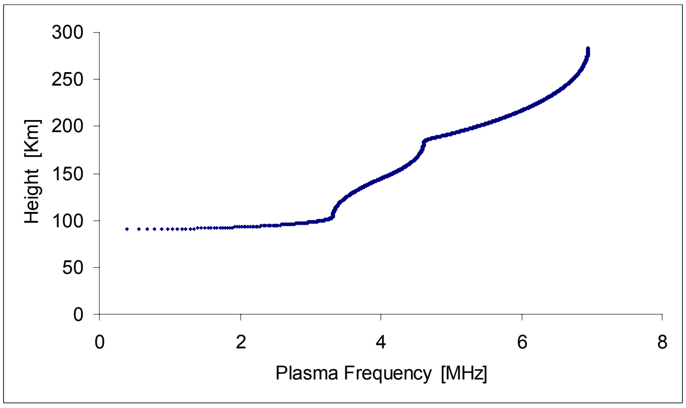
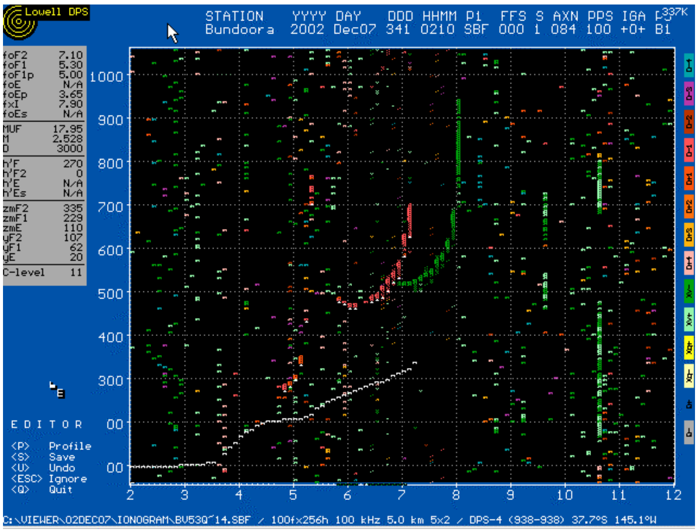


Fig. 2. The resultant ionospheric profile determined from the inversion of the VI ionogram trace in Fig. 1.profile

This example highlights the accuracy of the inversion technique. The technique homes in well to a QPS parameter solution of the ionosphere.

IV. TESTING THE INVERSION TECHNIQUE USING REAL VI IONOGRAM DATA

A typical daytime VI ionogram, recorded from the ionosonde stationed at La Trobe University, Bundoora, at time 0210 UT, 7th December 2002, is shown in Fig. 3. The red curve, in Fig 3 represents the Ordinary trace and the green curve represents the extraordinary trace. The white curve represents the shifted Chebyshev polynomial profile and the ARTIST layer parameters are also recorded.



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STATION    YYYY DAY    DDD HHMM P1  FFS S AXN PPS IGA P337K
Bundoora   2002 Dec07  341 0210 SBF 000 1 084 100 +0+ B1

foF2      7.10
foF1      5.30
foF1p     5.00
foE        N/A
foEp      3.65
fxI       7.90
foEs       N/A
MUF      17.95
M        2.528
D         3000
h'F        270
h'F2         0
h'E        N/A
h'Es       N/A
zmF2       335
zmF1       229
zmE        110
yF2        107
yF1         62
yE          20
C-level     11

C:\VIEWER\02DEC07\IONOGRAM\BV53Q˜14.SBF / 100fx256h 100 kHz 5.0 km 5x2 / DPS-4 (938-938) 37.7°S 145.1°W
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Fig. 3. A typical daytime VI ionogram.

The E layer echo trace is not distinguishable on this ionogram so the modeled E layer parameters are used to represent the E layer. Table II shows the data points chosen from the F1 and F2 layer echo traces on the VI ionogram, in Fig 3. Table II also shows the accuracy of the new inversion technique where the ($\Delta h'$) values show the difference between the calculated and measured group height values. The ($\Delta h'$ L) values show the difference between the calculated and measured group height values when using the ARTIST software [2].

TABLE II

INPUT DATA POINTS FROM THE VI IONOGRAM IN FIG 3.

Data point F1 layer	[MHz]	h' [km]	$\Delta h'$ [km]	$\Delta h'$ L. [km]
1	4.8	275.0	-22.6	-57.6
2	4.9	280.0	-30.9	-56.8
3	5.0	295.0	-33.8	-46.2
4	5.1	330.0	-24.4	-15.5
5	5.2	430.0	32.4	68.6
Data point F2 layer				
1	5.8	475.0	1.5	20.2
2	6.1	460.0	-0.3	12.8
3	6.5	480.0	6.1	29.7
4	7.0	590.0	-2.8	49.0

From Table II, the results clearly show that the new inversion technique described here produced the better results.

The computed layer parameters are:

$$foE_c = 3.2\,\text{MHz}\,,\ y_m E_c = 14\ \text{km}\,,\ r_{Ec} = 104 + r_0\,\text{km}$$

$$foF1_c = 4.61\,\text{MHz}\,,\ y_m F1_c = 77\ \text{km}\,,\ r_{F1_c} = 183 + r_0\,\text{km}$$

$$foF2_c = 6.95\text{MHz}\,,\ y_m F2_c = 130\ \text{km},\ r_{F2c} = 283 + r_0\,\text{km}$$

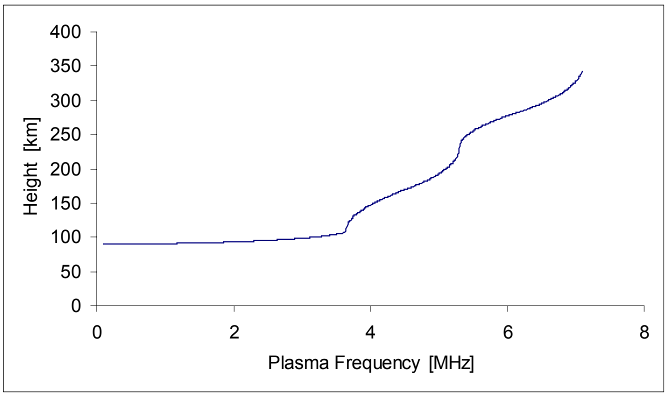

Fig. 4. The resultant ionospheric profile determined from the inversion of the VI ionogram trace in Fig. 3.

Figure 4 shows the resultant ionospheric profile having the layer parameters above.

The black curve in Fig 5 shows the synthesized VI ionogram resulting from the inversion technique described here. The green curve shows the corresponding VI trace when using the parameters given by the ARTIST software. The red dots show the data from the ordinary trace in Fig 3. Thus, from Table II and Fig 5, the layer parameters determined using the new inversion technique show a better agreement for both the F1 and F2 layer echo traces.

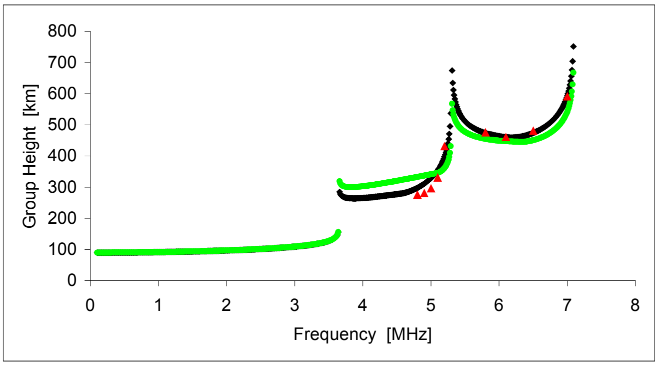

Fig. 5. Synthesized VI traces, where the black curve represents results using the new inversion method. The green curve shows the trace using the profile produced by ARTIST. The red triangles represent the real ionogram data.

V. THE NIGHTTIME IONOSPHERE

The nighttime ionosphere in general does not have an F1 layer thus only 2 layers represent the nighttime ionosphere. The QPS model is such that it fits a profile to each of the layers but the joining layer is chosen such that it smoothly attaches the E layer to the F layer and ignores the actual shape of the ionospheric profile. As a result, the nighttime ionosphere, which has a large valley region between the E layer and the F layer, is poorly fitted. This problem was solved, by [7], by adding a pseudo QPS layer. Another problem is that the F1 layer echo trace may not be a distinct layer. Reference [7] showed that by smoothly attaching the joining layer from the F2 layer to the peak of the F1 layer at a point just below the peak of the F1 layer that this can be solved. This then made a profile with out a distinct F1 layer. These methods can be readily implemented into the inversion scheme.

VI. CONCLUSION

This new inversion technique described here which determines the ionospheric profile from swept frequency vertical incidence ionograms is robust and in general more accurately determines the layer parameters of the Ionosphere than previous inversion methods. The inversion technique determines a QPS layer parameter representation of the ionosphere and unlike previous inversion techniques determines the layer parameters directly from the layer echo traces on the VI ionograms.

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